Anniversary
1963 – 1998

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

1998 Solutions

Gauss Contest

(Grade 7)

Part A

1. The value of $\frac{1998-998}{1000}$ is

(**A**) 1 (**B**) 1000 (**C**) 0.1 (**D**) 10 (**E**) 0.001

Solution

$\frac{1998-998}{1000}=\frac{1000}{1000}=1$ ANSWER: (A)

2. The number 4567 is tripled. The ones digit (units digit) in the resulting number is

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 3 (**E**) 1

Solution

If we wish to determine the units digit when we triple 4567, it is only necessary to triple 7 and take the units digit of the number 21.

The required digit is 1. ANSWER: (E)

3. If $S=6\times 10\ 000+5\times 1000+4\times 10+3\times 1$, what is S?

(**A**) 6543 (**B**) 65 043 (**C**) 65 431 (**D**) 65 403 (**E**) 60 541

Solution

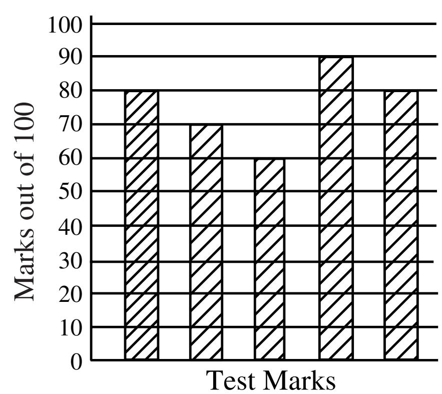
$S=60\ 000+5000+40+3$

$=65\ 043$ ANSWER: (B)

4. Jean writes five tests and achieves the marks shown on the graph. What is her average mark on these five tests?

(**A**) 74 (**B**) 76 (**C**) 70

(**D**) 64 (**E**) 79



Solution

Jean's average is $\frac{80+70+60+90+80}{5}=\frac{380}{5}=76$. ANSWER: (B)

5. If a machine produces 150 items in one minute, how many would it produce in 10 seconds?

(**A**) 10 (**B**) 15 (**C**) 20 (**D**) 25 (**E**) 30

Solution

Since 10 seconds represents one-sixth of a minute, the machine will produce $\frac{1}{6} \times 150$ or 25 items.

ANSWER: (D)

6. In the multiplication question, the sum of the digits in the four boxes is

(**A**) 13 (**B**) 12 (**C**) 27
(**D**) 9 (**E**) 22

$$\begin{array}{r} 879 \\ \times\, 492 \\ \hline \Box 758 \\ 7\Box 11 \\ 35\Box 6 \\ \hline 43\Box 468 \end{array}$$

Solution

Multiplying out,

$$\begin{array}{r} 879 \\ \times\, 492 \\ \hline 1758 \\ 7911 \\ 3516 \\ \hline 432468 \end{array}$$

The sum is $1+9+1+2=13$.

ANSWER: (A)

7. A rectangular field is 80 m long and 60 m wide. If fence posts are placed at the corners and are 10 m apart along the four sides of the field, how many posts are needed to completely fence the field?

(**A**) 24 (**B**) 26 (**C**) 28 (**D**) 30 (**E**) 32

Solution

There is 1 post on each corner making a total of 4 plus 7 along each of the two lengths and 5 along each of the two widths.
This gives a total of 28 posts.

ANSWER: (C)

8. Tuesday's high temperature was $4°$C warmer than that of Monday's. Wednesday's high temperature was $6°$C cooler than that of Monday's. If Tuesday's high temperature was $22°$C, what was Wednesday's high temperature?

(**A**) $20°$C (**B**) $24°$C (**C**) $12°$C (**D**) $32°$C (**E**) $16°$C

Solution

If Tuesday's temperature was $22°$C then Monday's high temperature was $18°$C.
Wednesday's temperature was $12°$C since it was $6°$C cooler than that of Monday's high temperature.

ANSWER: (C)

9. Two numbers have a sum of 32. If one of the numbers is -36, what is the other number?

(**A**) 68 (**B**) -4 (**C**) 4 (**D**) 72 (**E**) -68

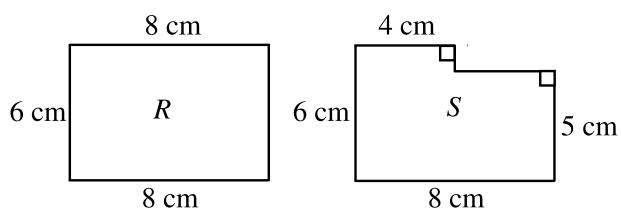
Solution
$68 + (-36) = 32$ ANSWER: (A)

10. At the waterpark, Bonnie and Wendy decided to race each other down a waterslide. Wendy won by 0.25 seconds. If Bonnie's time was exactly 7.80 seconds, how long did it take for Wendy to go down the slide?
(**A**) 7.80 seconds (**B**) 8.05 seconds (**C**) 7.55 seconds (**D**) 7.15 seconds (**E**) 7.50 seconds

Solution
If Wendy finished 0.25 seconds ahead of Bonnie and Bonnie took 7.80 seconds then Wendy took $7.80 - 0.25$ or 7.55 seconds. ANSWER: (C)

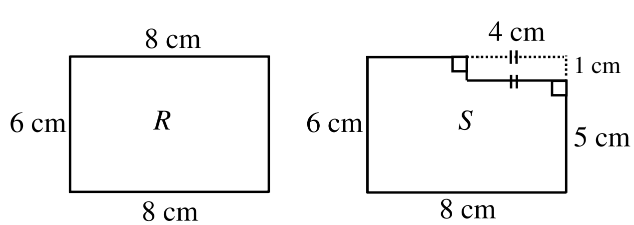
Part B

11. Kalyn cut rectangle *R* from a sheet of paper. A smaller rectangle is then cur from the large rectangle *R* to produce figure *S*. In comparing *R* to *S*



(**A**) the area and perimeter both decrease
(**B**) the area decreases and the perimeter increases
(**C**) the area and perimeter both increase
(**D**) the area increases and the perimeter decreases
(**E**) the area decreases and the perimeter stays the same

Solution
If figure *S* is cut out of rectangle *R* then *S* must be smaller in area.
If we compare perimeters, however, we find that the perimeter of figure *S* is identical to that of rectangle *R*.
The comparison of perimeter is not too difficult to see if we complete figure *S* as shown and compare lengths.
The perimeters of *R* and *S* are equal.



ANSWER: (E)

12. Steve plants ten trees every three minutes. If he continues planting at the same rate, how long will it take him to plant 2500 trees?

(**A**) $1\frac{1}{4}$ h (**B**) 3 h (**C**) 5 h (**D**) 10 h (**E**) $12\frac{1}{2}$ h

Solution 1

Since Steve plants ten trees every three minutes, he plants one tree every $\frac{3}{10}$ minute.

In order to plant 2500 trees, he will need $\frac{3}{10} \times 2500 = 750$ minutes or $\frac{750}{60} = 12\frac{1}{2}$ hours.

Solution 2

Since Steve plants ten trees every three minutes, he plants 200 trees per hour.

In order to plant 2500 trees, he will need $\frac{2500}{200} = 12\frac{1}{2}$ hours. ANSWER: (E)

13. The pattern of figures △●□▲○ is repeated in the sequence

△,●,□,▲,○,△,●,□,▲,○, ...

The 214th figure in the sequence is

(**A**) △ (**B**) ● (**C**) □ (**D**) ▲ (**E**) ○

Solution

Since the pattern repeats itself after every five figures, it begins again after 210 figures have been completed.

The 214th figure would be the fourth element in the sequence or ▲. ANSWER: (D)

14. A cube has a volume of 125 cm^3. What is the area of one face of the cube?

(**A**) 20 cm^2 (**B**) 25 cm^2 (**C**) $41\frac{2}{3} \text{ cm}^2$ (**D**) 5 cm^2 (**E**) 75 cm^2

Solution

If the volume of the cube is 125 cm^3, then the length, width and height are each 5 cm.

The area of one face is 5×5 or 25 cm^2. ANSWER: (B)

15. The diagram shows a magic square in which the sums of the numbers in any row, column or diagonal are equal. What is the value of n?

8		
9		5
4	n	

(**A**) 3 (**B**) 6 (**C**) 7
(**D**) 10 (**E**) 11

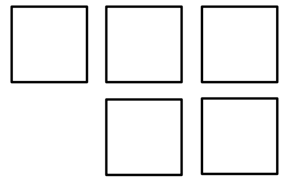
Solution

The 'magic' sum is $8 + 9 + 4 = 21$, so the centre square is 7.

If the centre square is 7, then the square on the lower right has 6 in it giving $4 + n + 6 = 21$.

Therefore $n = 11$. ANSWER: (E)

16. Each of the digits 3, 5, 6, 7, and 8 is placed one to a box in the diagram. If the two digit number is subtracted from the three digit number, what is the smallest difference?



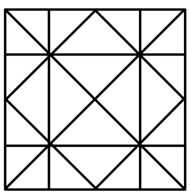
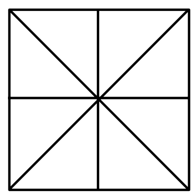
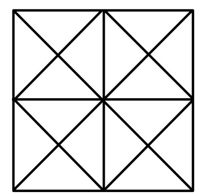
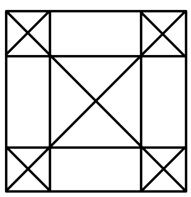
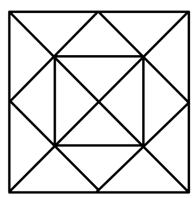
(**A**) 269 (**B**) 278 (**C**) 484
(**D**) 271 (**E**) 261

Solution

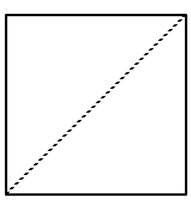
The smallest difference will be produced when the three digit number is as small as possible, that is 356, and the two digit number is as large as possible, that is 87.

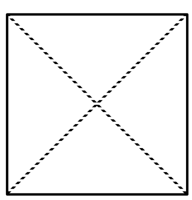
The smallest difference is $356 - 87 = 269$. ANSWER: (A)

17. Claire takes a square piece of paper and folds it in half four times without unfolding, making an isosceles right triangle each time. After unfolding the paper to form a square again, the creases on the paper would look like

(**A**)

(**B**)

(**C**)

(**D**)

(**E**)


Solution


Fold 1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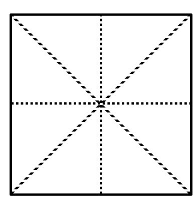
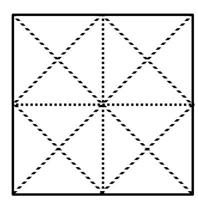

Fold 3


Fold 4

ANSWER: (C)

18. The letters of the word 'GAUSS' and the digits in the number '1998' are each cycled separately and then numbered as shown.

1. AUSSG 9981
2. USSGA 9819
3. SSGAU 8199

etc.

If the pattern continues in this way, what number will appear in front of GAUSS 1998?

(**A**) 4 (**B**) 5 (**C**) 9 (**D**) 16 (**E**) 20

Solution

Because the word 'GAUSS' has five letters in it, the numbers 5, 10, 15, 20, ... will appear beside this word. Similarly, the four digits of '1998' will have the numbers 4, 8, 12, 16, 20, 24, ... beside this number.

From this listing, we can see that the correct number is 20 which is the l.c.m. of 5 and 4.

ANSWER: (E)

19. Juan and Mary play a two-person game in which the winner gains 2 points and the loser loses 1 point. If Juan won exactly 3 games and Mary had a final score of 5 points, how many games did they play?

(**A**) 7 (**B**) 8 (**C**) 4 (**D**) 5 (**E**) 11

Solution

If Juan won 3 games then Mary lost 3 points so that she must have had 8 points before losing in order to have a final total of 5.

If Mary had 8 points before losing then she must have won 4 games.

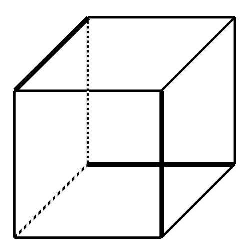
If Mary won 4 games and Juan won 3 games there was a total of 7 games. ANSWER: (A)

20. Each of the 12 edges of a cube is coloured either red or green. Every face of the cube has at least one red edge. What is the smallest number of red edges?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

Solution

If the heavy black lines represent the colour red, every face will have exactly one red edge. So the smallest number of red edges is 3.

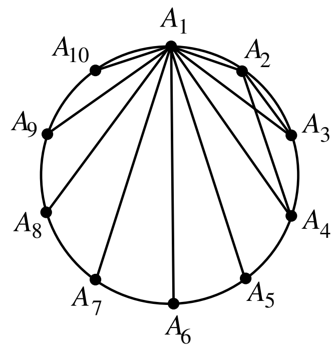


ANSWER: (B)

Part C

21. Ten points are spaced equally around a circle. How many different chords can be formed by joining any 2 of these points? (A chord is a straight line joining two points on the circumference of a circle.)
(**A**) 9 (**B**) 45 (**C**) 17 (**D**) 66 (**E**) 55

Solution
Space the ten points equally around the circle and label them $A_1, A_2, ..., A_{10}$ for convenience.
If we start with A_1 and join it to each of the other nine points, we will have 9 chords.
Similarly, we can join A_2 to each of the other 8 points. If we continue this process until we join A_9 to A_{10} we will have $9+8+7+6+5+4+3+2+1=45$ chords.



ANSWER: (B)

22. Each time a bar of soap is used, its volume decreases by 10%. What is the minimum number of times a new bar would have to be used so that less than one-half its volume remains?
(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

Solution

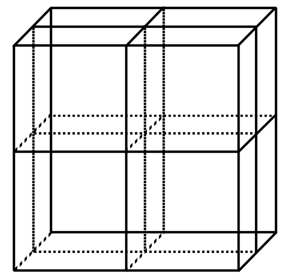
Number of Times Soap Used	Approximate Volume Remaining (as %)
1	0.9 or 90%
2	$(0.9)^2$ or 81%
3	$(0.9)^3$ or 72.9%
4	$(0.9)^4$ or 65.61%
5	$(0.9)^5$ or 59.1%
6	$(0.9)^6$ or 53.1%
7	$(0.9)^7$ or 47.8%

So if the soap is used 7 times the volume will be less than $\frac{1}{2}$ of the original volume.

NOTE: In essence, we are trying to find a positive integer x so that $(0.9)^x < 0.5$. The value of x can be found by using the $\boxed{y^x}$ button on your calculator where $y = 0.9$ and experimenting to find a value for x.

ANSWER: (C)

23. A cube measures $10\text{ cm} \times 10\text{ cm} \times 10\text{ cm}$. Three cuts are made parallel to the faces of the cube as shown creating eight separate solids which are then separated. What is the increase in the total surface area?

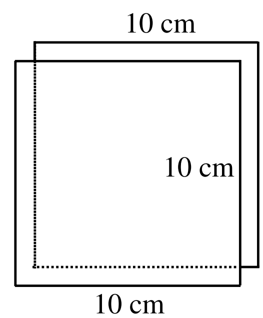


(**A**) 300 cm^2 (**B**) 800 cm^2 (**C**) 1200 cm^2
(**D**) 600 cm^2 (**E**) 0 cm^2

Solution

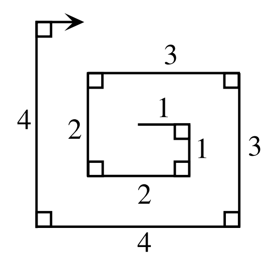
One cut increases the surface area by the equivalent of two $10\text{ cm} \times 10\text{ cm}$ squares or 200 cm^2.

Then the three cuts produce an increase in area of $3 \times 200\text{ cm}^2$ or 600 cm^2.



ANSWER: (D)

24. On a large piece of paper, Dana creates a “rectangular spiral” by drawing line segments of lengths, in cm, of 1, 1, 2, 2, 3, 3, 4, 4, ... as shown. Dana’s pen runs out of ink after the total of all the lengths he has drawn is 3000 cm. What is the length of the longest line segment that Dana draws?



(**A**) 38 (**B**) 39 (**C**) 54
(**D**) 55 (**E**) 30

Solution

The formula for the sum of the natural numbers from 1 to n is $\frac{(n)(n+1)}{2}$.

That is, $1+2+3+...+n=\frac{(n)(n+1)}{2}$.

(For example, $1+2+3+...+10=\frac{(10)(11)}{2}=55$.)

We can find the sum of a double series, like the one given, by doubling each side of the given formula.

We know $1+2+3+...+n=\frac{(n)(n+1)}{2}$.

If we double each side we get $2(1+2+...+n)=n(n+1)$.

So, $(1+1)+(2+2)+(3+3)+...+(n+n)=n(n+1)$.

In this question we want the value of n so that the following is true:

$(1+1)+(2+2)+(3+3)+...+(n+n) \le 3000$.

Or, if we use the formula $(n)(n+1) \le 3000$.

We would now like to find the largest value of n for which this is true.

The best way to start is by taking $\sqrt{3000} \doteq 54.7$ as a beginning point.
If we try $n = 54$, we find $(54)(55) = 2970 < 3000$ which is a correct estimate.
(If we try $n = 55$ we find $55(56) = 3080 > 3000$. So $n = 55$ is not acceptable.)
This means that $(1+1)+(2+2)+(3+3)+ ... +(54+54) = 2970$ so that the longest length that Dana completed was 54 cm. (If we had included the length 55 then we would have had a sum of 3 025 which is too large.) ANSWER: (C)

25. Two natural numbers, p and q, do not end in zero. The product of any pair, p and q, is a power of 10 (that is, 10, 100, 1000, 10 000, ...). If $p > q$, the last digit of $p - q$ cannot be
(**A**) 1 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 9

Solution
If the two natural numbers p and q do not end in zero themselves and if their product is a power of 10 then p must be of the form 5^n and q must be of the form 2^n.
This is true because $10 = 2 \times 5$ and $10^n = (2 \times 5)^n = 2^n \times 5^n$.
The possibilities for powers of two are 2, 4, 8, 16, 32, ... and for corresponding powers of five are 5, 25, 125, 625, 3125, ...
If we take their differences and look at the last digit of $p - q$ we find the following,

p	q	last digit of $p - q$
5	2	3
25	4	1
125	8	7
625	16	9
3125	32	3
15 625	64	1
⋮	⋮	7
		9
		⋮

and the pattern continues in groups of 4.

Thus, the last digit of $p - q$ cannot be 5. ANSWER: (C)